Multiobjective optimization: on the verge of numerical tractability

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Outline

- MO and Multple Criteria Decision Making (MCDM).

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- Large-scale (LS) MCDM $\Longrightarrow$ interactive LS MO.

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- Large-scale (LS) MCDM $\Longrightarrow$ interactive LS MO.
- LS MCDM: three case studies.
- Final comments.

LS MCDM

We say that we deal with an LS instance of MO when MO is not able to compute enough Pareto optimal **solutions** within the resource (time, computing power) budget for the respective MCDM to identify the most preferred decision **variant** (a solution to MO).

Here we present three approaches that can be applied to avoid such frustrating failures.

Approach I - Granular computing

Granular computing is an emerging computing paradigm of information processing that concerns the processing of complex information entities called "information granules", which arise in the process of data abstraction and derivation of knowledge from information or data. Generally speaking, information granules are collections of entities that usually originate at the numeric level and are arranged together due to their similarity, functional or physical adjacency, **indistinguishability**, coherency, or the like. At present, granular computing is more a theoretical perspective than a coherent set of methods or principles. As a theoretical perspective, it encourages an approach to data that recognizes and exploits the knowledge present in data at various levels of resolution or scales. In this sense, it encompasses all methods which provide flexibility and adaptability in the resolution at which knowledge or information is extracted and represented.

Source: *Wikipedia*

Granular computing in MO $\longrightarrow$ lower and upper bounds on criteria

All $f_l(x), \; l = 1, \ldots, k$, to be maximized.

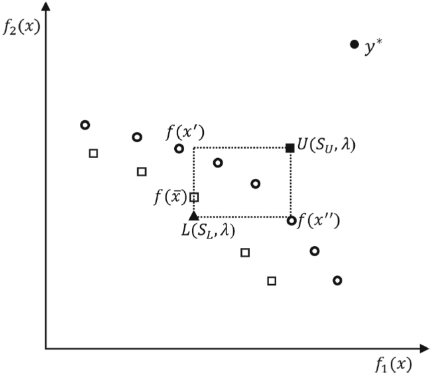
$$\min_{x \in X_0} \max_l \lambda_l((y_l^* - f_l(x)) + \rho e^k(y^* - f(x)))$$

$\lambda_l > 0$, $y_l^* > f_l(x)$ for any $x \in X_0, \; l = 1, \ldots, k$, $\rho > 0$ and "small".

It depends on λ which x is derived.

Given λ, $x(\lambda)$ is the *implicit solution*.

Granular computing in MO $\implies$ lower and upper bounds on criteria values of implicit solutions



Source: *Miroforidis, J. (2021), Bounds on efficient outcomes for large-scale cardinality-constrained Markowitz problems. Journal of Global Optimization, 80, 617–634.*

Mean-variance portfolio selection problem with the portfolio cardinality constraint

$$f_1(x) := -x^T Qx \ \Rightarrow \max \quad \text{(negative variance)}$$

$$f_2(x) := r^T x \ \Rightarrow \max \quad \text{(expected return)}$$

$$x \in X_0 = \{x \mid x \geq 0,\ \sum_{t=1}^{n} x_t = 1, card(x) \leq C\},$$

where

Q : $n \times n$ matrix of instrument covariance,

r : vector of instrument expected returns,

C : an integer.

Mean-variance portfolio selection problem with the portfolio cardinality constraint

Data from the New York Stock Exchange were collected for 600 companies, for the period from July 2001 until June 2018, with no missing quotes. The data set in the format analogous to the Beasley problems from OR Library is available at:

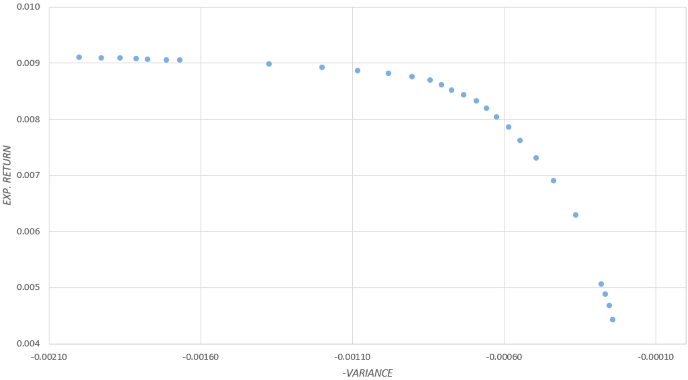
- http://www.ibspan.waw.pl/ kaliszew/ JKMP_portfolio_problems/JKMP_600/JKMP_600.txt.

More problems, up to 1000 instruments, at:

- https://repod.icm.edu.pl/dataset.xhtml?persistentId =doi:10.18150/CZYLOV

Software: Gurobi 8.1.1 for Windows 10 (x64).
Platform: CPU Intel Core i7-7700HQ, 16 GB RAM.

Mean-variance portfolio selection problem with the portfolio cardinality constraint



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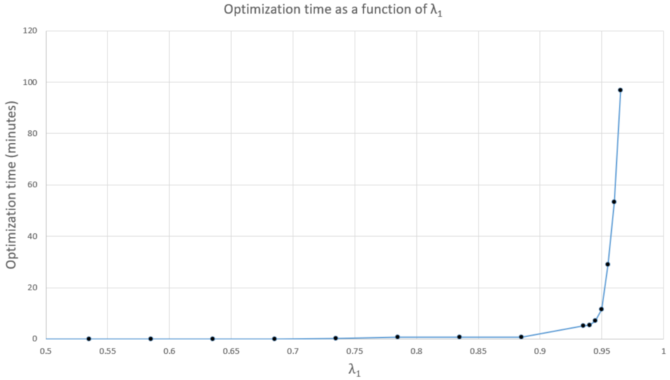

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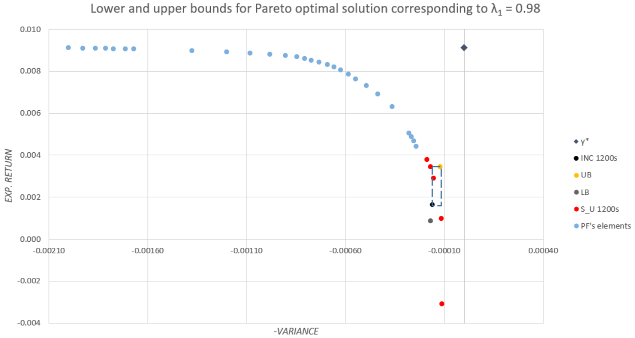
Mean-variance portfolio selection problem with the portfolio cardinality constraint



Total computing time: 4200 sec.

Gurobi did not provided optimal solution in 7500 sec. (MIP GAP% = 5.78% is an useless information in the MO context).

$$-0.0001686 \leq f_1(x(\lambda)) \leq -0.0001112,$$

$$0.0008468 \leq f_2(x(\lambda)) \leq 0.0034285.$$

Source: *Miroforidis, J. (2021), Bounds on ...*

Granular computing in MO $\longrightarrow$ lower and upper bounds on criteria

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- Kaliszewski, I., Out of the mist - towards decision-maker-friendly Multiple Criteria Decision Making support. European Journal of Operational Research, 293–307, 293-07, 2004.
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Granular computing in MO $\longrightarrow$ lower and upper bounds on criteria

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Approach II – Abolish the Dogmas

- Dogma 1: Mean-variance portfolio selection problem with the portfolio cardinality constraint (MVCC) is a hard quadratic programming problem.
- Dogma 2: Research in quadratic programming to provide more effective solving methods is highly motivated by needs of the financial industry.

A Rudimentary Approximation of Pareto optimal portfolios in MVCC problem

MVCC:

$$f_1(x) := -x^T Qx \;\Rightarrow \max \quad \text{(negative variance)}$$

$$f_2(x) := r^T x \;\Rightarrow \max \quad \text{(expected return)}$$

$$x \in X_0 = \{x \mid x \geq 0,\; \sum_{i=1}^{n} x_i = 1, card(x) \leq C\},$$

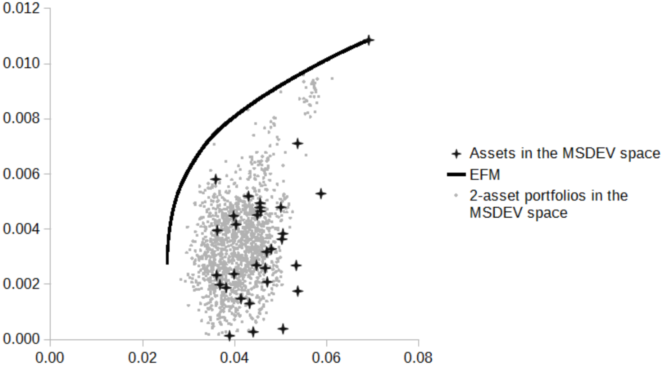
$$\alpha x^i + (1-\alpha)x^j = x,\; 0 \leq \alpha \leq 1,$$

If $\sum_{t=1}^{n} x_t^i = 1$ and $\sum_{t=1}^{n} x_i^j = 1$, then $\sum_{t=1}^{n} x_t = 1$.

(x^i feasible and x^j feasible $\Longrightarrow$) x feasible.

A Rudimentary Approximation of Pareto optimal portfolios in MVCC problem

Data: Beasley 31 asset portfolio problem published in
`http://people.brunel.ac.uk/~mastjjb/jeb/info.html`



Source: *Juszczuk P., Kaliszewski I., Miroforidis J., Podkopaev D., Expected mean return – standard deviation efficient frontier approximation with low cardinality portfolios in the presence of the risk-free assets. International Transactions of Operational Research, 30, 2395-2414, 2023.*

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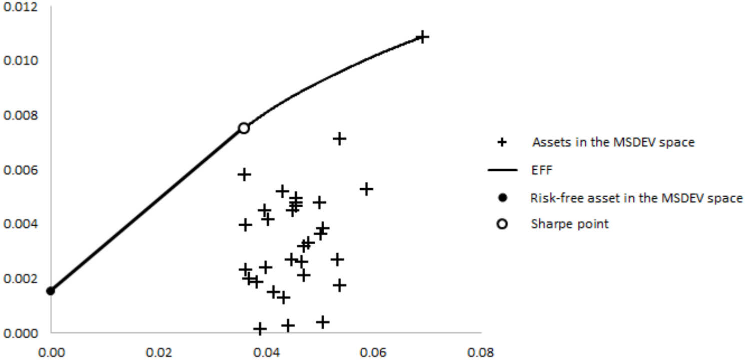

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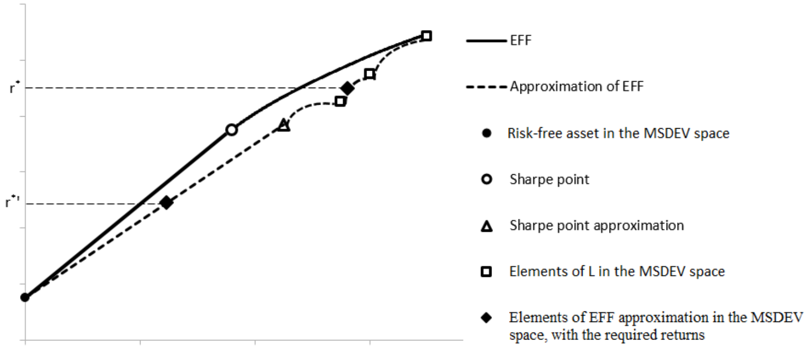

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A Rudimentary Approximation of Pareto optimal portfolios in MVCC problem

Approximation errors:

	Iteration 1		Iteration 4	
Data set	Avg. (%)	Max. (%)	Avg. (%)	Max. (%)
Beasley 31	0.35	3.42	0.03	0.24
Beasley 85	11.43	19.22	0.38	1.54
Beasley 89	4.56	11.27	0.30	2.32
Beasley 98	10.28	27.34	0.75	2.33
Beasley 225	0.97	4.97	0.88	4.97
JKMP 300	8.81	27.60	1.26	9.43
JKMP 350	12.03	33.37	1.59	4.90
JKMP 400	19.51	46.55	2.37	8.11
JKMP 500	28.60	49.25	2.66	6.23
JKMP 600	15.50	22.24	1.79	2.81

Source: *Juszczuk P. et al., Expected mean return . . .* .

A Rudimentary Approximation of Pareto optimal portfolios in MVCC problem

Portfolio cardinalities:

Data set	Approximate efficient portfolios		Efficient portfolios	
	Avg.	Max.	Avg.	Max.
Beasley 31	3.2	4	3	4
Beasley 85	3.3	5	4.5	9
Beasley 89	5.3	7	6.6	11
Beasley 98	6.4	9	9.1	19
Beasley 225	4.1	6	3.9	6
JKMP 300	8.8	9	8.7	15
JKMP 350	6.3	9	10.3	19
JKMP 400	6.8	11	9.4	19
JKMP 500	7.0	9	11.6	26
JKMP 600	6.8	8	9.8	15

Source: *Juszczuk P. et al., Expected mean return . . .* .

A Rudimentary Approximation of Pareto optimal portfolios in MVCC problem

- Juszczuk P., Kaliszewski I., Miroforidis J., Podkopaev D., **Expected mean return – standard deviation efficient frontier approximation with low cardinality portfolios in the presence of the risk-free assets. International Transactions of Operational Research**, 30, 2395-2414, 2023.
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Approach III – Exact and heuristic optimization hybrides

Combining exact optimization methods with heuristics

– the case of Intensity Modulated Radiotherapy.

Intensity Modulated Radiotherapy (IMRT)

1. Kill malicious cells (tumor) by irradiation.
2. Protect surrounding organs.

Source: *Author's photo, University Hospital Torrecardenas, Almeria, Spain*.

Intensity Modulated Radiotherapy (IMRT)

Source: *Author's photo, Maria Sklodowska-Curie National Research Institute of Oncology, Warsaw, Poland*.

Intensity Modulated Radiotherapy (IMRT)

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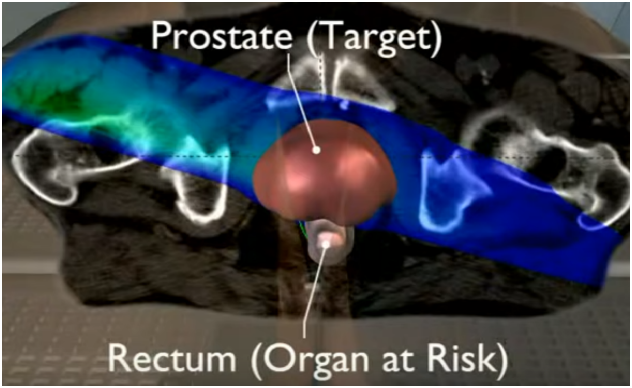
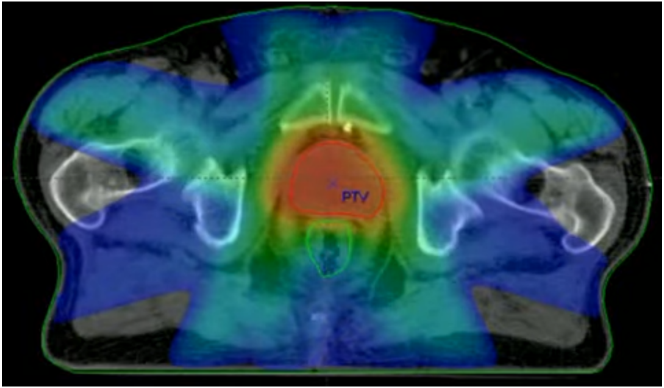
IMRT



Source: *Varian Medical Systems, https://www.youtube.com/watch?v=o2-5GyPTxLg*.

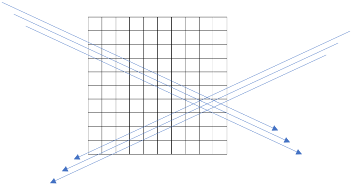
IMRT



Source: *Varian Medical Systems, https://www.youtube.com/watch?v=o2-5GyPTxLg*.

IMRT

Physical model:



30265 beamlets (number of variables) interacting with 94647 voxels ($\sim$ number of constraints)

Optimization model:

3D voxels, beams, beamlets,

Physically motivated (statistical) constrains (max/avg radiation doses),

Physically motivated (statistical) objective functions (uniform malicious cells coverage),

Biologically motivated objective functions.

IMRT

Equivalent Uniform Dose (EUD) – biologically motivated objective function (multiplicative form).

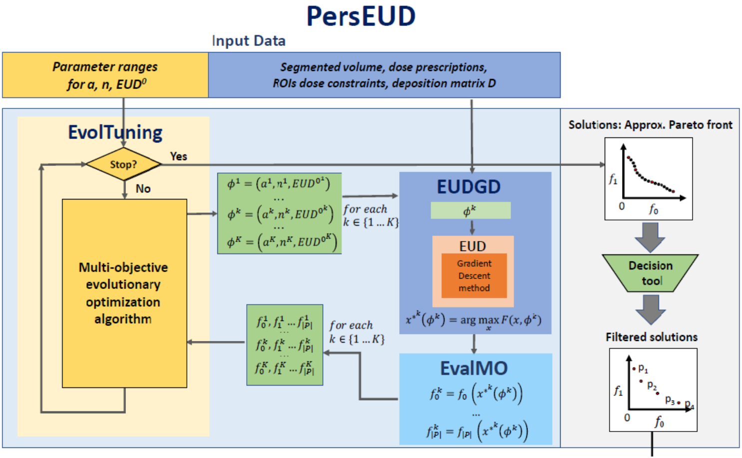
$$F(x,\phi) = \prod_{t\in T} \frac{1}{1+\left(\frac{EUD_t^0}{gEUD_t(x,a_t)}\right)^{n_t}} \cdot \prod_{r\in R} \frac{1}{1+\left(\frac{gEUD_r(x,a_r)}{EUD_r^0}\right)^{n_r}}$$

Unconstrained optimization: excellent uniform dose coverage, violations of physical constraints.

Physically motivated optimization model: takes care of feasibility of soft constraints:

$$\min_X \,(f_0, f_1, \ldots, f_{|P|})$$

IMRT



Search space: *parameters of EUD f.*

Search space: X

Source: *Moreno et al., IMRT planning . . . .*

IMRT

Optimization methods

For evolutionary optimization - MOEA/D algorithm (can be easily replaced)

For the EUD optimization – two in-house implementations of the Gradient Descent method:

1. on GPUS: CUDA C software for NVIDIA-based GPUs
2. on multicore computers: OpenMP C software for multicore CPUs.

Multicore hardware of the High-Performance Cluster of the SAL (Supercomputacíon–Algoritmos) group, the University of Almeria, Spain in two configurations:

– AMD EPYC 7302 (32 CPU cores), 512 GB of DDR4 RAM, and two NVIDIA Tesla V100 (32 GB).

– two Intel Xeon E5-2620v3 (12 CPU cores), 64 GB of DDR3 RAM, and two NVIDIA Kepler K80 (12 GB).

These two implementations can be used interchangeably or even in parallel to exploit all available resources.

Cooperating units

Department of Medicine Physics, The Maria Sklodowska-Curie National Research Institute of Oncology, Warsaw. Poland.

Faculty of Electrical Engineering, Warsaw University of Technology, Warsaw, Poland.

Computer Science Department, Supercomputing-Algorithms group, University of Almeria, Almeria, Spain.

Systems Research Institute of the Polish Academy of Science, Warsaw, Poland.

Approach III – Exact and heuristic optimization hybrides – IMRT

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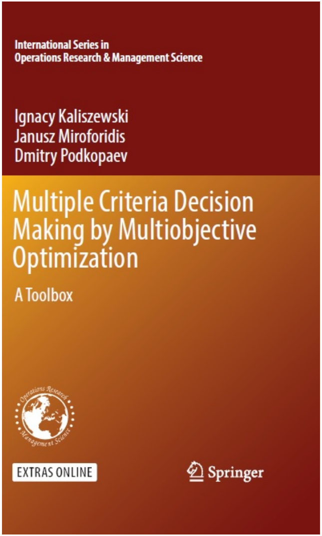
THANK YOU!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